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Spontaneous Supersymmetric Generation of an Indeterminate Mass Scale and a Possible Light Sterile Neutrino

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Abstract

If a global continuous symmetry of a supersymmetric field theory is spontaneously broken while preserving the supersymmetry, the resulting theory has a massless superfield. One of its two bosonic degrees of freedom is the familiar phase rotation of the usual massless Nambu-Goldstone boson, but the other is a scale transformation. An indeterminate mass scale is thus generated. In the fermion sector, a seesaw texture appears which may be suitable for a possible light sterile neutrino. This feature persists even after the gauging of the continuous symmetry or the breaking of the supersymmetry to resolve the aforementioned mass-scale ambiguity.

The physical mass scales of a renormalizable quantum field theory are expected to be determined by its explicit parameters, such as in quantum electrodynamics, or by the structure of its interactions, such as in quantum chromodynamics. In either case, even if the vacuum has nontrivial topology, the physical mass scales of the theory are uniquely determined. Consider the textbook example of the U(1) scalar model, with the potential

$$V_1 = m^2\phi^*\phi + \frac{1}{2}\lambda(\phi^*\phi)^2. \tag{1}$$

If $m^2 < 0$, then the global U(1) symmetry is broken by the vacuum expectation value (VEV) of the scalar field ϕ, *i.e.*

$$\langle\phi\rangle \equiv v = \sqrt{\frac{-m^2}{\lambda}}\, e^{i\theta}. \tag{2}$$

Redefining

$$\phi \equiv v + e^{i\theta}\left(\frac{H + i\xi}{\sqrt{2}}\right), \tag{3}$$

the physical theory is then given by

$$V_2 = \frac{1}{2}m_H^2 H^2 + \frac{1}{2}m_H\sqrt{\lambda}H(H^2+\xi^2) + \frac{1}{8}\lambda(H^2+\xi^2)^2, \tag{4}$$

where $m_H^2 = -2m^2$, *i.e.* the mass scale of this theory is still determined by the input m^2. As is well-known, the massless Nambu-Goldstone boson ξ is a manifestation [1] of the rotational degree of freedom $e^{i\theta}$ of Eq. (2).

Consider now a supersymmetric U(1) model. Assume the spontaneous breakdown of the global U(1) symmetry, but not the supersymmetry [2]. The Goldstone theorem [3] requires the existence of a massless Nambu-Goldstone boson, which must then be part of a massless superfield, *i.e.* there must exist a massless complex scalar field together with its massless fermionic superpartner. One of its two bosonic degrees of freedom is the analog of $e^{i\theta}$, but the other is a scale transformation e^{α}, where α is a real parameter. It is clear that a model with just one VEV cannot have this property, because its ground state is obviously not

invariant under the transformation $v \to ve^{\alpha}$. On the other hand, if there are two VEVs, it may become possible to have $v_1 \to v_1 e^{\alpha}$ and $v_2 \to v_2 e^{-\alpha}$ so that their product $v_1 v_2$ remains unchanged. As shown below, that is exactly what happens in the simplest realization of this phenomenon. Since $|v_1|^2 + |v_2|^2$ is unconstrained by the explicit parameters of such a model, an indeterminate mass scale is generated.

The simplest model which exhibits the behavior under consideration has three superfields: ϕ_1 and ϕ_2 transform oppositely under U(1) and χ is trivial under it. The most general superpotential is given by

$$W = \mu\phi_1\phi_2 + f\phi_1\phi_2\chi + \frac{1}{2}m\chi^2 + \frac{1}{3}h\chi^3, \tag{5}$$

from which the following scalar potential is obtained:

$$V = |\mu + f\chi|^2 \left(|\phi_1|^2 + |\phi_2|^2\right) + |f\phi_1\phi_2 + m\chi + h\chi^2|^2. \tag{6}$$

There are three $V = 0$ solutions: (1) $\phi_1 = \phi_2 = \chi = 0$; (2) $\phi_1 = \phi_2 = 0$, $\chi = -m/h$; and (3) $\chi = -\mu/f$, $\phi_1\phi_2 = m\mu/f^2 - h\mu^2/f^3$. Whereas U(1) is unbroken in the first two cases, it is spontaneously broken in the third, resulting in the existence of "flat directions" [4].

Let $\phi_{1,2}$ and χ be shifted to $v_{1,2} + \phi_{1,2}$ and $u + \chi$ in Eq. (6), where

$$u = -\frac{\mu}{f} \tag{7}$$

and

$$v_1 v_2 = \frac{m\mu}{f^2} - \frac{h\mu^2}{f^3}, \tag{8}$$

then V yields the following mass terms:

$$\begin{aligned} V_2 &= \left[|f|^2\left(|v_1|^2 + |v_2|^2\right) + |m + 2hu|^2\right]|\chi|^2 + |f|^2|v_2\phi_1 + v_1\phi_2|^2 \\ &+ \left[f^*(m + 2hu)(v_2^*\phi_1^* + v_1^*\phi_2^*)\chi + h.c.\right]. \end{aligned} \tag{9}$$

This shows clearly that the linear combination

$$\zeta \equiv \frac{-v_1^* \phi_1 + v_2^* \phi_2}{\sqrt{|v_1|^2 + |v_2|^2}} \tag{10}$$

is massless, whereas

$$\eta \equiv \frac{v_2 \phi_1 + v_1 \phi_2}{\sqrt{|v_1|^2 + |v_2|^2}} \tag{11}$$

and χ are massive with their mass-squared matrix given by

$$\mathcal{M}^2 = \left[\begin{array}{cc} |A|^2 & A^* B \\ AB^* & |A|^2 + |B|^2 \end{array} \right], \tag{12}$$

where

$$A = f\sqrt{|v_1|^2 + |v_2|^2}, \quad B = m + 2hu. \tag{13}$$

The constraint of Eq. (8) applies only to the product $v_1 v_2$, hence A of Eq. (13) is an indeterminate parameter.

Unlike the usual Nambu-Goldstone boson, ζ of Eq. (10) is a <u>complex</u> scalar field. One of its two degrees of freedom corresponds to having

$$v_1 \to v_1 e^{i\theta}, \quad v_2 \to v_2 e^{-i\theta} \tag{14}$$

in analogy to the familiar invariance of the ground state with respect to a phase rotation, but the other is a scale transformation, *i.e.*

$$v_1 \to v_1 e^{\alpha}, \quad v_2 \to v_2 e^{-\alpha}. \tag{15}$$

Hence the individual values of $|v_1|$ and $|v_2|$ are not separately determined. This is the consequence of the spontaneous breakdown of a global continuous symmetry together with the assumed preservation of the supersymmetry. It is also easily shown that the fermion partners of η and χ have the mass matrix

$$\mathcal{M} = \left[\begin{array}{cc} 0 & A \\ A & B \end{array} \right], \tag{16}$$

hence $\mathcal{M}^\dagger\mathcal{M} = \mathcal{M}^2$ of Eq. (12) as expected.

The superpotential W of Eq. (5) has 4 parameters: f, h, μ, and m. The spontaneously broken theory has instead 5 parameters: f, h, u, v_1, and v_2. Whereas u and v_1v_2 are constrained by Eqs. (7) and (8), $|v_1|^2+|v_2|^2$ is not. The new superpotential is then given by

$$\begin{aligned} W' &= f\sqrt{|v_1|^2+|v_2|^2}\eta\chi + \frac{1}{2}(m+2hu)\chi^2 + \frac{1}{3}h\chi^3 \\ &+ \frac{f\chi}{|v_1|^2+|v_2|^2}\left[v_1^*v_2^*\eta^2 + (|v_2|^2-|v_1|^2)\eta\zeta - v_1v_2\zeta^2\right]. \end{aligned} \tag{17}$$

If it is not known that W is the antecedent of W', one may worry that the massless superfield ζ would not stay massless in the presence of interactions. As it is, because of the Goldstone theorem, all such higher-order effects do in fact cancel and ζ is indeed massless. I have checked this explicitly to one-loop order, and have ascertained that the cancellation works for arbitrary values of $|v_1|^2+|v_2|^2$. I note also that this phenomenon occurs in general whenever a global continuous symmetry is reduced in rank by one spontaneously while preserving the supersymmetry.

Since $|v_1|^2+|v_2|^2$ is not fixed by the input parameters f, h, μ, and m of W, the scale of the spontaneous breakdown of the global U(1) symmetry is arbitrary, subject only to the algebraic inequality $|v_1|^2+|v_2|^2 \geq 2|v_1v_2|$. This is an unusual phenomenon which may be relevant to cosmology if supersymmetry is a good description of fundamental interactions in the early Universe. It allows for the possibility of different domains, not just of phase, but of scale. On the other hand, this ambiguity of scale may be just a curiosity and is naturally eliminated in a realistic theory. There are two ways, as discussed below.

One way is to promote the global U(1) symmetry to a local U(1) symmetry [5], so that there are now gauge interactions which generate an extra term in the scalar potential:

$$V_D = \frac{1}{2}g^2(\phi_1^*\phi_1 - \phi_2^*\phi_2)^2, \tag{18}$$

thus enforcing the equality $|v_1| = |v_2|$. In this case, the scalar field $\sqrt{2}Re\zeta$ acquires a nonzero mass from V_D, whereas $\sqrt{2}Im\zeta$ remains massless. The latter is of course the familiar would-be Nambu-Goldstone boson which gets absorbed by the U(1) gauge boson to render the latter massive [6]. Both it and $\sqrt{2}Re\zeta$ have the mass $2g|v|$, as are their fermionic partners.

The other is to break the supersymmetry which is certainly necessary phenomenologically. As shown below, this would also fix v_1 and v_2 separately, even if the supersymmetry breaking parameter is very small, as long as it is nonzero, *i.e.* the scale-invariant supersymmetric ground state is inherently <u>unstable</u>.

Let V of Eq. (6) be supplemented with the soft supersymmetry breaking term $a|\phi_1|^2$, where $a < 0$. The minimum of V is now shifted:

$$\langle\chi\rangle = u = -\frac{\mu}{f} + b, \tag{19}$$

and

$$\langle\phi_1\phi_2\rangle = v_1 v_2 = -\frac{mu}{f} - \frac{hu^2}{f} + c, \tag{20}$$

where

$$\begin{aligned} v_1(a + f^2 b^2) + v_2 f^2 c &= 0, & (21) \\ v_2 b^2 + v_1 c &= 0, & (22) \\ (v_1^2 + v_2^2) f b + (m + 2hu) c &= 0. & (23) \end{aligned}$$

For $a = 0$, it is clear that the only solution is $b = c = 0$. For $a \neq 0$, it becomes

$$b = \frac{v_1}{f}\sqrt{\frac{-a}{v_1^2 - v_2^2}}, \quad c = \frac{v_1 v_2 a}{f^2(v_1^2 - v_2^2)}, \tag{24}$$

and

$$\frac{v_1^2}{v_2^2} - 1 = \frac{-(m + 2hu)^2 a}{f^4 (v_1^2 + v_2^2)^2}. \tag{25}$$

In the above, all parameters have been assumed real for simplicity. It is clear that the mass-scale ambiguity of the supersymmetric theory is now resolved.

Whereas the spontaneous supersymmetric generation of an indeterminate mass scale and its resolution are not new ideas [7], the preceding discussion serves to point out an interesting new byproduct. The fermion mass matrix $\mathcal{M}$ of Eq. (16) remains the same in the case of gauging the U(1) symmetry, and is approximately the same in the case of softly broken supersymmetry. It has a seesaw texture in that one diagonal entry is zero. This is a generic result from the Yukawa coupling of three superfields of charges $+1$, -1, and 0. If the parameters of the theory are such that $|A| << |B|$ in Eq. (16), one mass eigenvalue is naturally light, *i.e.* $|A|^2/|B|$, by the well-known seesaw mechanism [8]. This does require some fine tuning, *i.e.* $fm \simeq h\mu$. However, since it involves only parameters of the superpotential, the condition is stable against radiative corrections.

The appearance of a light Majorana fermion with no standard-model interactions may be regarded as a sterile neutrino. It may be relevant to the current experimental situation of neutrino oscillations, where positive signals are being claimed in atmospheric [9], solar [10], and accelerator [11] data. With three known doublet neutrinos, it is difficult to accommodate all three sets of data with sensitivity to three very different mass scales. Hence a fourth singlet neutrino is needed [12].

In conclusion, although it is possible to generate an indeterminate mass scale from the spontaneous breakdown of a global continuous symmetry of a supersymmetric field theory, the resulting scale-invariant ground state is inherently unstable with respect to any soft supersymmetry breaking. It also does not happen if the global symmetry is made local. However, an interesting generic feature occurs in the fermion sector, where a light sterile neutrino may appear.

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